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Isomorphic Property and Some Operations of Picture Fuzzy Graphs

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Abstract: In this paper picture fuzzy graph has shown more advantage in handling vagueness and uncertainty than fuzzy set. We have applied concept of picture fuzzy set to fuzzy set. Picture fuzzy graph is generalization of fuzzy graph. The concept of picture fuzzy graph has discussed here. In this paper complement and Ring sum operation of picture fuzzy graphs has discussed isomorphic property of picture fuzzy graphs. We have used picture Fuzzy graph play important role in representation of many uncertain decision making problem in daily life since Picture fuzzy set is an efficient mathematical model to deal with uncertain real life problems, e.g. number theory, coding theory, cryptography, computer science, operation research, also we are using certain concepts of bipolar fuzzy directed hypergraphs Finally we have proved the Ring sum of two picture fuzzy graphs is also a picture fuzzy graph.

Keywords: Fuzzy Graph, Isomorphism, Picture Fuzzy Graph, Complement of Fuzzy Graph

1. Introduction

Graphs do not represent all the systems properly due to uncertainty or vagueness of the parameter of system. In such case it is natural to deal with the uncertainty using the method of fuzzy theory introduced by zadeh. In 1994 Rosenfeld [4] introduce fuzzy graphs andseveral fuzzy analogs of graph theoretic concepts. Since several other formulation of fuzzy graph problems have appeared in literature.

The Picture fuzzy set is an efficient mathematical model to deal with uncertain real life problems, in which fuzzy set mayfail to reveal satisfactory results. Picture fuzzy graph is an extension of the classical fuzzy set. It can work very efficiently in uncertain circumstances which involve more answers like yes, no, abstain and refusal. Fuzzy graph has used to model many decision making problem in uncertain environment. A number of generalizations of fuzzy graphs have been introduced to deal the uncertainty of the real life problem. Picture fuzzy graph would be prominent research direction for modeling the uncertain optimization problems.

In 2019Cen Zuo [3] all provides a picture fuzzy social network, a social unit is represented by node, a social unit has some good, neutral and bad activity. In this study complement of picture fuzzy graph, Ring sum operation of picture fuzzy graphs has discussed. Isomorphic property of picture fuzzy graphs based on picture fuzzy relation has mentioned. Further we have proved the Ring sum of two picture fuzzy graphs is also a picture fuzzy graph. In 2017 Akram,

M., Luqman, A [1] proved Certain concepts of bipolar fuzzy directed hypergraphs. In 2008 A. Nagoor Gani and J. Malarvizhi [2] proved Isomorphism on Fuzzy Graphs. In 1994 Johan N. Mordeson and Chang-Shyh Peng [5] gave operations on fuzzy graph. In 2000 J. N. Mordeson P. S. Nair [6] introduce Fuzzy Graphs and Fuzzy Hyper graphs. In 1965 L. A. Zadeh [7] introduce Fuzzy sets. In 2002 M. S. Sunitha andA. VijayaKumar [8] introduce Complement of fuzzy graph. In 2011 Mehidi Eatemadi, Ali Etemadi &

Mohammad-Mehdi Ebadzadeh [9] gave idea about Finding theIsomorphic graph with the use of algorithms based on DNA. In 2017M. Akramand R. Akmal [10] gave Intuitionistic Fuzzy Graph Structures. In1975R. T. Yeh and S. Y. Banh [11] gave Fuzzy relations, fuzzy graphs and their applicationsto clustering analysis. In 2018 S. Samanta and B. Sarkar [12] gave Representation of completions by generalized fuzzy graphs. In 2018 Sarwar, M., Akram, M., Alshehri, N. O. [13] introduce a new method to decision-making with fuzzy competition hyper graphs. In 2015 Sing P. [14] introduce Correlation coefficients for Picture fuzzy set. In 2007 Y. Vaishnaw and A. S. Ranadive [15] gave isomorphism between fuzzy graphs.

2. Preliminary

Definition 2.1 [4] Let G be a graph whose vertex set is V,σ be a fuzzy sub set of V and μ be a fuzzy sub set of V×V then the pair (σ, μ) is called fuzzy graph if

$$\mu(u,v) \le \sigma(u) \wedge \sigma(v), \forall\, u,v \in V$$

Definition2.2 [11] A fuzzy graph (σ, μ) is said to be complete if

$$\mu(u,v) = \sigma(u) \wedge \sigma(v) = \mu(u,v), \forall\, u,v \in V$$

Definition 2.3 [11] Consider the fuzzy graph $G_1: (\sigma_1, \mu_1)$ and $G_2: (\sigma_2, \mu_2)$ with $\sigma_1^* = V_1$ and $\sigma_2^* = V_2$ are isomorphism between two fuzzy graphsG_1 andG_2 is a bijective mapping

$$h: V_1 \to V_2 \sigma_1(u) = \sigma_2\big(h(u)\big) \forall\, u \in V$$

and $\forall\, h(u) \in V_1$ And $\mu_1(u,v) = \mu_2\big(h(u), h(v)\big)\ \forall\, u,v \in V_1$ we write in this case$G_1 \simeq G_2$.

3. Main Results

Definition 3.1 A pair $G = (V\ E)$ is called a picture fuzzy graph on $G^* = (V^* E^*)$ where$V = (\mu_V, \sigma_V, \rho_V)$ is a picture fuzzy set on $V^*$and $V = (\mu_E, \sigma_E, \rho_E)$ is picture fuzzy set on $E^* \subseteq V^* \times V^*$such that for each arc $uv \in E^*$

$$M_E(uv) \le \big(\mu_V(u) \wedge \mu_V(v)\big), \sigma_E(uv)$$
$$\le (\sigma_V(u) \wedge \sigma_V(v)), \rho_E(uv)$$
$$= (\rho_V(u) \vee \rho_V(v)$$

Example 3.1

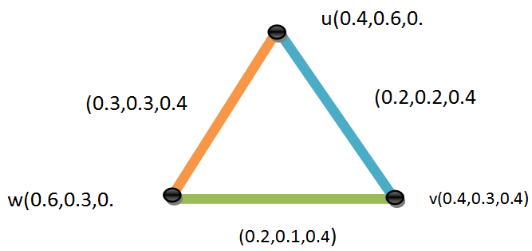

Figure 1. *Picture Fuzzy Graph.*

Definition 3.2: A picture fuzzy graph $G = (V\ E)$ is said to be complete picture fuzzy graph if

$$\mu_E(uv)\big(\mu_V(u) \wedge \mu_V(v)\big), \sigma_E(uv) = (\sigma_V(u) \wedge\ \sigma_V(v)), \rho_E(uv) = (\rho_V(u) \vee \rho_V(v)$$

Where $V = (\mu_V \sigma_V \rho_V)$ is a picture fuzzy set on V* and $V = (\mu_E \sigma_E \rho_E)$ is picture fuzzy set on $E^* \subseteq V^* \times V^*$such that forall $u, v \in V^*$

Definition 3.3 A picture fuzzy graph $G = (V\ E)$ is said to be strong picture fuzzy graph if

$$\mu_E(uv) = \big(\mu_V(u) \wedge \mu_V(v)\big), \sigma_E(uv) =(\sigma_V(u) \wedge\ \sigma_V(v)), \rho_E(uv)\ = (\rho_V(u) \vee \rho_V(v)$$

Where $V = (\mu_V \sigma_V \rho_V)$is a picture fuzzy set on V*

And $V = \big(\mu_E \sigma_E \rho_E\big)$ picture is fuzzy set on $E^* \subseteq V^* \times V^*$for all $uv \in E^*$

Definition 3.4 The complement of a picture fuzzy graph $G = (V, E)$ is a picture fuzzy graph $G' = (V', E')$if and only if as follows-

$$\mu'_V = \mu_V, \sigma'_V = \sigma_V, \rho'_V = \rho_V, \mu'_E(uv) = \big(\mu_V(u) \wedge \mu_V(v)\big) - \mu_E(uv),$$
$$\sigma'_E(uv) =\ \sigma_V(u) \wedge\ \sigma_V(v)) - \sigma_E(uv), \rho'_E(uv)$$
$$= (\rho_V(u) \vee \rho_V(v) - \rho_E(uv)\ \forall u, v \in V^*$$

Proposition3.1 The complement of a picture fuzzy graph $G = (V, E)$ is also a picture fuzzy graph.
Proof:

$$\mu'_V = \mu_V, \sigma'_V = \sigma_V, \rho'_V = \rho_V$$
$$\mu'_E(uv) = \big(\mu_V(u) \wedge \mu_V(v)\big) - \mu_E(uv) \le \big(\mu_V(u) \wedge \mu_V(v)\big) = \big(\mu'_V(u) \wedge \mu'_V(v)\big),$$
$$\sigma'_E(uv) =(\sigma_V(u) \wedge\ \sigma_V(v)) - \sigma_E(uv)$$
$$\le (\ \sigma_V(u) \wedge\ \sigma_V(v)) = (\ \sigma'_V(u) \wedge\ \sigma'_V(v)),$$

$$\rho'_E(uv) = (\rho_V(u) \vee \rho_V(v) - \rho'_E(uv) \leq (\rho_V(u) \vee \rho_V(v) = (\rho'_V(u) \vee \rho'_V(v)$$

Hence complement of a picture fuzzy graph $G = (V, E)$ is also a picture fuzzy graph.

Definition 3.5 Suppose $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two picture fuzzy graphs of $G^*_1 = (V^*_1, E^*_1)$ and $G^*_2 = (V^*_2, E^*_2)$ respectively then $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are said to be isomorphic if a bijective mappingh: $V_1 \longrightarrow V_2$ such that

$$\mu_{A_1(u_1)} = \mu_{A_2(u_2)}, \sigma_{A_1}(u_1) = \sigma_{A_2}(h(u_1))$$

$$\rho_{A_1}(u_1) = \rho_{A_2}(h(u_1)) \text{ and}$$

$$\mu_{B_1}(u_1, v_1) = \mu_{B_2}(h(u_1), h(v_1))$$

$$\sigma_{B_1}(u_1, v_1) = \sigma_{B_2}(h(u_1), h(v_1))$$

$$\rho_{B_1}(u_1, v_1) = \rho_{B_2}(h(u_1), h(v_1)),$$

Definition 3.6Suppose $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two picture fuzzy graphs of $G^*_1 = (V^*_1, E^*_1)$ and $G^*_2 = (V^*_2, E^*_2)$ respectively thenjoin $G_1 + G_2$ of two picture fuzzy graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is defined as $G = (V, E)$ Where $V = (\mu_A, \sigma_A, \rho_A)$ is picture fuzzy set on $V^* = V^*_1 \cup V^*_2$ and $E = (\mu_B, \sigma_B, \rho_B)$ is an another picture fuzzyset on $E^* = E^*_1 \cup E^*_2 \cup E'$ (E' represents the set of all arcs joining the vertices ofV_1 and V_2).

$$\mu_V(u) = (\mu_{V_1} + \mu_{V_2})u = \begin{cases} \mu_{V_1}(u) \text{ if } u \in V_1^* \\ \mu_{V_2}(u) \text{ if } u \in V_2^* \\ \mu_{V_1}(u) \wedge \mu_{V_2}(u) \text{ if } u \in V_1^* \cap V_2^* \end{cases}$$

$$\sigma_V(u) = (\mu_{V_1} + \mu_{V_2})u = \begin{cases} \sigma_{V_1}(u) \text{ if } u \in V_1^* \\ \sigma_{V_2}(u) \text{ if } u \in V_2^* \\ \sigma_{V_1}(u) \wedge \sigma_{V_2}(u) \text{ if } u \in V_1^* \cap V_2^* \end{cases}$$

$$\rho_V(u) = (\rho_{V_1} + \rho_{V_2})u = \begin{cases} \rho_{V_1}(u) \text{ if } u \in V_1^* \\ \rho_{V_2}(u) \text{ if } u \in V_2^* \\ \rho_{V_1}(u) \vee \rho_{V_2}(u) \text{ if } u \in V_1^* \cap V_2^* \end{cases}$$

$$\mu_E(uv) = (\mu_{E_1} + \mu_{E_2})uv = \begin{cases} \mu_{E_1}(uv) \text{ if } uv \in E_1^* \\ \mu_{E_2}(uv) \text{ if } uv \in E_2^* \\ \mu_{E_1}(uv) \wedge \mu_{E_2}(uv) \text{ if } uv \in E_1^* \cap E_2^* \end{cases}$$

$$\sigma_E(uv) = (\sigma_{E_1} + \sigma_{E_2})uv = \begin{cases} \sigma_{E_1}(uv) \text{ if } uv \in E_1^* \\ \sigma_{E_2}(uv) \text{ if } uv \in E_2^* \\ \sigma_{E_1}(uv) \wedge \sigma_{E_2}(uv) \text{ if } uv \in E_1^* \cap E_2^* \end{cases}$$

$$\rho_E(uv) = (\rho_{E_1} + \rho_{E_2})uv = \begin{cases} \rho_{E_1}(uv) \text{ if } uv \in E_1^* \\ \rho_{E_2}(uv) \text{ if } uv \in E_2^* \\ \rho_{E_1}(uv) \vee \rho_{E_2}(uv) \text{ if } uv \in E_1^* \cap E_2^* \end{cases}$$

$$\mu_E(uv) = (\mu_{E_1} + \mu_{E_2})uv = \mu_{E_1}(uv) \vee \mu_{E_2}(uv) \text{ if } uv \in E'$$

$$\sigma_E(uv) = (\sigma_{E_1} + \sigma_{E_2})uv = \sigma_{E_1}(uv) \vee \sigma_{E_2}(uv) \text{ if } uv \in E'$$

$$\rho_E(uv) = (\rho_{E_1} + \rho_{E_2})uv = \rho_{E_1}(uv) \wedge \rho_{E_2}(uv) \text{ if } uv \in E'$$

Definition 3.7 Suppose $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two picture fuzzy graphs of $G^*_1 = (V^*_1, E^*_1)$ and $G^*_2 = (V^*_2, E^*_2)$ respectively thenUnion $G_1 \cup G_2$ of two picture fuzzy graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is defined as $G = (V, E)$ Where $V = (\mu_A, \sigma_A, \rho_A)$ is picture fuzzy set on $V^* = V^*_1 \cup V^*_2$ and $E = (\mu_B, \sigma_B, \rho_B)$ is an another picture product fuzzyset on $E^* = E^*_1 \cup E^*_2$

$$\mu_V(u) = (\mu_{V_1} \cup \mu_{V_2})u = \begin{cases} \mu_{V_1}(u) \text{ if } u \in V_1^* \\ \mu_{V_2}(u) \text{ if } u \in V_2^* \\ \mu_{V_1}(u) \wedge \mu_{V_2}(u) \text{ if } u \in V_1^* \cap V_2^* \end{cases}$$

$$\sigma_V(u) = (\mu_{V_1} \cup \mu_{V_2})u = \begin{cases} \sigma_{V_1}(u) \text{ if } u \in V_1^* \\ \sigma_{V_2}(u) \text{ if } u \in V_2^* \\ \sigma_{V_1}(u) \wedge \sigma_{V_2}(u) \text{ if } u \in V_1^* \cap V_2^* \end{cases}$$

$$\rho_V(u) = (\rho_{V_1} \cup \rho_{V_2})u = \begin{cases} \rho_{V_1}(u) \text{ if } u \in V_1^* \\ \rho_{V_2}(u) \text{ if } u \in V_2^* \\ \rho_{V_1}(u) \wedge \rho_{V_2}(u) \text{ if } u \in V_1^* \cap V_2^* \end{cases}$$

$$\mu_E(uv) = (\mu_{E_1} \cup \mu_{E_2})uv = \begin{cases} \mu_{E_1}(uv) \text{ if } uv \in E_1^* \\ \mu_{E_2}(uv) \text{ if } uv \in E_2^* \\ \mu_{E_1}(uv) \wedge \mu_{E_2}(uv) \text{ if } uv \in E_1^* \cap E_2^* \end{cases}$$

$$\sigma_E(uv) = (\sigma_{E_1} \cup \sigma_{E_2})uv = \begin{cases} \sigma_{E_1}(uv) \text{ if } uv \in E_1^* \\ \sigma_{E_2}(uv) \text{ if } uv \in E_2^* \\ \sigma_{E_1}(uv) \wedge \sigma_{E_2}(uv) \text{ if } uv \in E_1^* \cap E_2^* \end{cases}$$

$$\rho_E(uv) = (\rho_{E_1} \cup \rho_{E_2})uv = \begin{cases} \rho_{E_1}(uv) \text{ if } uv \in E_1^* \\ \rho_{E_2}(uv) \text{ if } uv \in E_2^* \\ \rho_{E_1}(uv) \wedge \rho_{E_2}(uv) \text{ if } uv \in E_1^* \cap E_2^* \end{cases}$$

Definition 3.8Suppose $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two picture fuzzy graphs of $G^*_1 = (V^*_1, E^*_1)$ and $G^*_2 = (V^*_2, E^*_2)$ respectively thenRing sum$G_1 \oplus G_2$ of two picture fuzzy graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is defined as $G = (V, E)$ where $V = (\mu_A, \sigma_A, \rho_A)$ is picture fuzzy set on $V^* = V^*_1 \cup V^*_2$ and $E = (\mu_B, \sigma_B, \rho_B)$ is an another picture fuzzyset on $E^* = E^*_1 \cup E^*_2$

$$\mu_V(u) = (\mu_{V_1} \oplus \mu_{V_2})u = \begin{cases} \mu_{V_1}(u) \text{ if } u \in V_1^* \\ \mu_{V_2}(u) \text{ if } u \in V_2^* \\ \mu_{V_1}(u) \wedge \mu_{V_2}(u) \text{ if } u \in V_1^* \cap V_2^* \end{cases}$$

$$\sigma_V(u) = (\mu_{V_1} \oplus \mu_{V_2})u = \begin{cases} \sigma_{V_1}(u) \text{ if } u \in V_1^* \\ \sigma_{V_2}(u) \text{ if } u \in V_2^* \\ \sigma_{V_1}(u) \wedge \sigma_{V_2}(u) \text{ if } u \in V_1^* \cap V_2^* \end{cases}$$

$$\rho_V(u) = (\rho_{V_1} \oplus \rho_{V_2})u = \begin{cases} \rho_{V_1}(u) \text{ if } u \in V_1^* \\ \rho_{V_2}(u) \text{ if } u \in V_2^* \\ \rho_{V_1}(u) \wedge \rho_{V_2}(u) \text{ if } u \in V_1^* \cap V_2^* \end{cases}$$

$$\mu_E(uv) = (\mu_{E_1} \oplus \mu_{E_2})uv = \begin{cases} \mu_{E_1}(uv) \text{ if } uv \in E_1^* \\ \mu_{E_2}(uv) \text{ if } uv \in E_2^* \\ 0 \text{ otherwise} \end{cases}$$

$$\sigma_E(uv) = (\sigma_{E_1} \oplus \sigma_{E_2})uv = \begin{cases} \sigma_{E_1}(uv) \text{ if } uv \in E_1^* \\ \sigma_{E_2}(uv) \text{ if } uv \in E_2^* \\ 0 \text{ otherwise} \end{cases}$$

$$\rho_E(uv) = (\rho_{E_1} \oplus \rho_{E_2})uv = \begin{cases} \rho_{E_1}(uv) \text{ if } uv \in E_1^* \\ \rho_{E_2}(uv) \text{ if } uv \in E_2^* \\ 0 \text{ otherwise} \end{cases}$$

Theorem3.1 Suppose $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two picture fuzzy graphs of $G^*_1 = (V^*_1, E^*_1)$ and $G^*_2 = (V^*_2, E^*_2)$ respectively thenRing sum$G_1 \oplus G_2$ of two picture fuzzy graphs$G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is also a picture fuzzy graphs.

Proof: We have to show that the Ring sum$G_1 \oplus G_2$ of two picture fuzzy graphs$G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is also a picture fuzzy graphs. For thisit is sufficient to show that

$$\mu_E(uv) = (\mu_{E_1} \oplus \mu_{E_2})uv$$

$$\leq (\mu_{V_1} \oplus \mu_{V_2})u \wedge (\mu_{V_1} \oplus \mu_{V_2})v),$$

$$\sigma_E(uv) = (\sigma_{E_1} \oplus \sigma_{E_2})uv \leq (\sigma_{V_1} \oplus \sigma_{V_2})u \wedge (\sigma_{V_1} \oplus \sigma_{V_2})v) \text{ and}$$

$$\rho_E(uv) = (\rho_{E_1} \oplus \rho_{E_2})uv \leq (\rho_{V_1} \oplus \rho_{V_2})u \vee (\rho_{V_1} \oplus \rho_{V_2})v) \text{ in different cases.}$$

If $uv \in E_1^* - E_2^*$ and $u, v \in V_1^* - V_2^*$ then

$$\mu_E(uv) = (\mu_{E_1} \oplus \mu_{E_2})uv = \mu_{E_1}(uv) \le \mu_{V_1}(u) \wedge \mu_{V_1}(u)$$

$$= [(\mu_{V_1} \cup \mu_{V_2})u \wedge (\mu_{V_1} \cup \mu_{V_2})v]$$

$$= [(\mu_{V_1} \oplus \mu_{V_2})u \wedge (\mu_{V_1} \oplus \mu_{V_2})v]$$

$$\sigma_E(uv) = (\sigma_{E_1} \oplus \sigma_{E_2})uv = \sigma_{E_1}(uv)$$

$$\le \sigma_{V_1}(u) \wedge \sigma_{V_1}(u)$$

$$= [(\sigma_{V_1} \cup \sigma_{V_2})u \wedge (\sigma_{E_1} \cup \sigma_{E_2})v]$$

$$= [(\sigma_{E_1} \oplus \sigma_{E_2})u \wedge (\sigma_{V_1} \oplus \sigma_{V_2})v]$$

$$\rho_E(uv) = (\rho_{E_1} \oplus \rho_{E_2})uv = \rho_{E_1}(uv)$$

$$\le \rho_{V_1}(u) \wedge \rho_{V_1}(u)$$

$$= [(\rho_{V_1} \cup \rho_{V_2})u \wedge (\rho_{V_1} \cup \rho_{V_2})v] = [(\rho_{V_1} \oplus \rho_{V_2})u \wedge (\rho_{V_1} \oplus \rho_{V_2})v]$$

If $uv \in E_1^* - E_2^*$ and $u \in V_1^* - V_2^*$ $v \in V_1^* \cap V_2^*$ then

$$\mu_E(uv) = (\mu_{E_1} \oplus \mu_{E_2})uv = \mu_{E_1}(uv) \le \mu_{V_1}(u) \wedge \mu_{V_1}(u)$$

$$= [(\mu_{V_1} \cup \mu_{V_2})u \wedge (\mu_{V_1} \wedge \mu_{V_2})v],$$

$$= [(\mu_{V_1} \oplus \mu_{V_2})u \wedge (\mu_{V_1} \oplus \mu_{V_2})v]$$

$$\sigma_E(uv) = (\sigma_{E_1} \oplus \sigma_{E_2})uv = \sigma_{E_1}(uv) \le \sigma_{V_1}(u) \wedge \sigma_{V_1}(u)$$

$$= [(\sigma_{V_1} \cup \sigma_{V_2})u \wedge (\sigma_{V_1} \times \sigma_{V_2})v] = [(\sigma_{V_1} \oplus \sigma_{V_2})u \wedge (\sigma_{V_1} \oplus \sigma_{V_2})v]$$

$$\rho_E(uv) = (\rho_{E_1} \oplus \rho_{E_2})uv = \rho_{E_1}(uv) \le \rho_{V_1}(u) \wedge \rho_{V_1}(u)$$

$$= [(\rho_{V_1} \cup \rho_{V_2})u \wedge (\rho_{V_1} \times \rho_{V_2})v]$$

$$= [(\rho_{V_1} \oplus \rho_{V_2})u \wedge (\rho_{V_1} \oplus \rho_{V_2})v]$$

If $uv \in E_1^* - E_2^*$ and $u, v \in V_1^* \cap V_2^*$ then

$$\mu_E(uv) = (\mu_{E_1} \oplus \mu_{E_2})uv = \mu_{E_1}(uv)$$

$$\le \mu_{V_1}(u) \wedge \mu_{V_2}(v) \wedge \mu_{V_1}(u) \wedge \mu_{V_2}(v)$$

$$= [(\mu_{V_1} \cup \mu_{V_2})u \wedge (\mu_{V_1} \cup \mu_{V_2})v]$$

$$= [(\mu_{V_1} \oplus \mu_{V_2})u \wedge (\mu_{V_1} \oplus \mu_{V_2})v]$$

$$\sigma_E(uv) = (\sigma_{E_1} \oplus \sigma_{E_2})uv = \sigma_{E_1}(uv)$$

$$\le \sigma_{V_1}(u) \wedge \sigma_{V_2}(v) \wedge \sigma_{V_1}(u) \wedge \sigma_{V_2}(v)$$

$$= [(\sigma_{V_1} \cup \sigma_{V_2})u \wedge (\sigma_{V_1} \cup \sigma_{V_2})v]$$

$$= [(\sigma_{V_1} \oplus \sigma_{V_2})u \wedge (\sigma_{V_1} \oplus \sigma_{V_2})v]$$

$$\rho_E(uv) = (\rho_{E_1} \oplus \rho_{E_2})uv = \rho_{E_1}(uv) \le \rho_{V_1}(u) \wedge \rho_{V_2}(v) \wedge \rho_{V_1}(u) \wedge \rho_{V_2}(v)$$

$$= [(\rho_{V_1} \cup \rho_{V_2})u \wedge (\rho_{V_1} \cup \rho_{V_2})v] = [(\rho_{V_1} \oplus \rho_{V_2})u \wedge (\rho_{V_1} \oplus \rho_{V_2})v]$$

Likewise we can prove the theorem in following conditions

If $uv \in E_2^* - E_1^*$ and $u, v \in V_2^* - V_1^*$

If $uv \in E_2^* - E_1^*$ and $u \in V_2^* - V_1^*$ $v \in V_1^* \cap V_2^*$,

If $uv \in E_2^* - E_1^*$ and $u, v \in V_1^* \cap V_2^*$, and If $uv \in E_2^* \cap E_1^*$ and $u, v \in V_1^* \cap V_2^*$

Remark:- Suppose $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two picture fuzzy graphs of $G^*_1 = (V^*_1, E^*_1)$ and $G^*_2 = (V^*_2, E^*_2)$ respectively and Ring sum $G_1 \oplus G_2$ is picture fuzzy graph of $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ then we have

$$\overline{\overline{(\mu_{E_1} \oplus \mu_{E_2})u}} = \overline{(\mu_{E_1} \oplus \mu_{E_2})u} = (\mu_{E_1} \oplus \mu_{E_2})u,$$

$$\overline{\overline{(\sigma_{E_1} \oplus \sigma_{E_2})u}} = (\sigma_{E_1} \oplus \sigma_{E_2})u, \overline{\overline{(\rho_{E_1} \oplus \rho_{E_2})u}} = (\rho_{E_1} \oplus \rho_{E_2})u$$

$$\text{And} \overline{\overline{(\mu_{E_1} \oplus \mu_{E_2})uv}} = \mu_{V_1}(u) \wedge \mu_{V_2}(v) - \overline{(\mu_{E_1} \oplus \mu_{E_2})uv}$$

$$= \mu_{V_1}(u) \wedge \mu_{V_2}(v) - \{\mu_{V_1}(u) \wedge \mu_{V_2}(v) - (\mu_{E_1} \oplus \mu_{E_2})uv\} = (\mu_{E_1} \oplus \mu_{E_2})uv$$

And similarly

$$\overline{\overline{(\sigma_{E_1} \oplus \sigma_{E_2})uv}} = (\sigma_{E_1} \oplus \sigma_{E_2})uv, \overline{\overline{(\rho_{E_1} \oplus \rho_{E_2})uv}} = (\rho_{E_1} \oplus \rho_{E_2})uv$$

Theorem 3.2 Suppose $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two picture fuzzy graphs of $G^*_1 = (V^*_1, E^*_1)$ and $G^*_2 = (V^*_2, E^*_2)$ respectively ($E^*_1 \cap E^*_2 \neq \emptyset$)then

$$\overline{(G_1 \oplus G_2)} \approx \overline{G_1} + \overline{G_2}$$

Proof:- Given that $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$are two picture fuzzy graphs of classicalgraphs $G^*_1 = (V^*_1, E^*_1)$and $G^*_2 = (V^*_2, E^*_2)$ respectively and $I: V_1 \oplus V_2 \to V_1 + V_2$ be the identity mapping, then it is sufficient to show that

$$\overline{(\mu_{V_1} \oplus \mu_{V_2})u} = (\overline{\mu_{V_1}} + \overline{\mu_{V_2}})u, \overline{(\sigma_{V_1} \oplus \sigma_{V_2})u}$$

$$= (\overline{\sigma_{V_1}} + \overline{\sigma_{V_2}})u, \overline{(\rho_{V_1} \oplus \rho_{V_2})u}$$

$$= (\overline{\rho_{V_1}} + \overline{\rho_{V_2}})u$$

$$\text{And} \overline{(\mu_{E_1} \oplus \mu_{E_2})uv} = (\overline{\mu_{E_1}} + \overline{\mu_{E_2}})uv, \overline{(\sigma_{E_1} \oplus \sigma_{E_2})uv} = (\overline{\sigma_{E_1}} + \overline{\sigma_{E_2}})uv$$

$$\overline{(\rho_{E_1} \oplus \rho_{E_2})uv} = (\overline{\rho_{E_1}} + \overline{\rho_{E_2}})uv$$

$$\text{Now} \overline{(\mu_{V_1} \oplus \mu_{V_2})u} = (\mu_{V_1} \oplus \mu_{V_2})u = (\mu_{V_1} \cup \mu_{V_2})u = (\overline{\mu_{V_1}} + \overline{\mu_{V_2}})u$$

$$\overline{(\sigma_{V_1} \oplus \sigma_{V_2})u} = (\sigma_{V_1} \oplus \sigma_{V_2})u$$

$$= (\sigma_{V_1} \cup \sigma_{V_2})u = (\overline{\sigma_{V_1}} + \overline{\sigma_{V_2}})u \ \overline{(\rho_{V_1} \oplus \rho_{V_2})u} = (\rho_{V_1} \oplus \rho_{V_2})u$$

$$= (\rho_{V_1} \cup \rho_{V_2})u = (\overline{\rho_{V_1}} + \overline{\rho_{V_2}})u$$

$$\text{And} \ \overline{(\mu_{E_1} \oplus \mu_{E_2})uv} = (\overline{\mu_{V_1} \oplus \mu_{V_2}})u \wedge (\overline{\mu_{V_1} \oplus \mu_{V_2}})u - (\mu_{E_1} \oplus \mu_{E_2})uv$$

$$= (\mu_{V_1} + \mu_{V_2})u \wedge (\mu_{V_1} + \mu_{V_2})u - (\mu_{E_1} \oplus \mu_{E_2})uv =$$

$$\begin{cases} (\mu_{V_1} + \mu_{V_2})u \wedge (\mu_{V_1} + \mu_{V_2})u - (\mu_{E_1})uv \text{ if } uv \in E^*_1 - E^*_2 \\ (\mu_{V_1} + \mu_{V_2})u \wedge (\mu_{V_1} + \mu_{V_2})u - (\mu_{E_2})uv \text{if } uv \in E^*_2 - E^*_1 \\ (\mu_{V_1} + \mu_{V_2})u \wedge (\mu_{V_1} + \mu_{V_2})u \text{ therwise} \end{cases}$$

Next we illustrate different cases like
Case-1 when $uv \in E'$ i.e. $u \in V_1$ and $v \in V_2$

$$\overline{(\mu_{E_1} \oplus \mu_{E_2})uv} = (\overline{\mu_{V_1} \oplus \mu_{V_2}})u \wedge (\overline{\mu_{V_1} \oplus \mu_{V_2}})u - (\mu_{E_1} \oplus \mu_{E_2})uv$$

$$= (\mu_{V_1} + \mu_{V_2})u \wedge (\mu_{V_1} + \mu_{V_2})u - 0 = [\overline{\mu_{V_1}}(u) \wedge \overline{\mu_{V_2}}(v)] = [\overline{\mu_{E_1}} + \overline{\mu_{E_2}}]uv$$

$$= (\overline{\sigma_{V_1} \oplus \sigma_{V_2}})u \wedge (\overline{\sigma_{V_1} \oplus \sigma_{V_2}})u - (\sigma_{E_1} \oplus \sigma_{E_2})uv$$

$$= (\sigma_{V_1} + \sigma_{V_2})u \wedge (\sigma_{V_1} + \sigma_{V_2})u - 0$$

$$= [\overline{\sigma_{V_1}}(u) \wedge \overline{\sigma_{V_2}}(v)] = [\overline{\sigma_{E_1}} + \overline{\sigma_{E_2}}]uv$$

$$= \left(\overline{\rho_{V_1} \oplus \rho_{V_2}}\right)u \wedge \left(\overline{\rho_{V_1} \oplus \rho_{V_2}}\right)u$$

$$-\left(\rho_{E_1} \oplus \rho_{E_2}\right)uv$$

$$= \left(\rho_{V_1} + \rho_{V_2}\right)u \wedge \left(\rho_{V_1} + \rho_{V_2}\right)u - 0 = \left[\overline{\rho_{V_1}}(u) \wedge \overline{\rho_{V_2}}(v)\right] = \left[\overline{\rho_{E_1}} + \overline{\rho_{E_2}}\right]uv$$

And similarly we can prove $\overline{\left(\mu_{E_1} \oplus \mu_{E_2}\right)}uv = (\overline{\mu_1} + \overline{\mu_1})uv$ in following different cases

1. When$uv \in E^*_1 - E^*_2$and $u, v \in V^*_1 - V^*_2$
2. When$uv \in E^*_1 - E^*_2$and $u, v \in V^*_1 \cap V^*_2$
3. When $uv \in E^*_1 - E^*_2$ and$u \epsilon V^*_1 \cap V^*_2, v \in V^*_1 \cap V^*_2$
4. When$uv \in E'$ i.e. $u \in V_1$and $v \in V_2$,
5. When $uv \in E^*_2 - E^*_1$and $u, v \in V^*_2 - V^*_1$
6. When$uv \in E^*_2 - E^*_1$ and $u, v \in V^*_1 \cap V^*_2$
7. When$uv \in E^*_2 - E^*_1$ and $u \epsilon V^*_1 \cap V^*_2$ $v \in V^*_1 \cap V^*_2$

Theorem 3.2 Suppose $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two picture fuzzy graphs of $G^*_1 = (V^*_1, E^*_1)$ and $G^*_2 = (V^*_2, E^*_2)$ respectively $(E^*_1 \cap E^*_2 \neq \emptyset)$

Then$(\overline{G_1 + G_2}) = \overline{G_1} \oplus \overline{G_2}$

Proof: Since $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two picture fuzzy graphs of $G^*_1 = (V^*_1, E^*_1)$ and $G^*_2 = (V^*_2, E^*_2)$ respectively ($E^*_1 \cap E^*_2 \neq \emptyset$) then

Case I when $uv \in E^*_1 - E^*_2$and $u, v \in V^*_1 - V^*_2$

$$\left(\overline{\mu_{V_1} + \mu_{V_2}}\right)u = \left(\mu_{V_1} \oplus \mu_{V_2}\right)u$$

$$= \mu_{V_1}(u) = \overline{\mu_{V_1}}(u) = (\overline{\mu_{V_1}} \oplus \overline{\mu_{V_2}})u$$

$$\left(\overline{\sigma_{V_1} + \sigma_{V_2}}\right)u = \left(\sigma_{V_1} \oplus \sigma_{V_2}\right)u = \sigma_{V_1}(u) = \overline{\sigma_{V_1}}(u) = (\overline{\sigma_{V_1}} \oplus \overline{\sigma_{V_2}})u$$

$$\left(\overline{\rho_{V_1} + \rho_{V_2}}\right)u = \left(\rho_{V_1} \oplus \rho_{V_2}\right)u = \rho_{V_1}(u) = \overline{\rho_{V_1}}(u) = (\overline{\rho_{V_1}} \oplus \overline{\rho_{V_2}})u$$

$$\text{And}\overline{\left(\mu_{E_1} + \mu_{E_2}\right)uv} = \left(\overline{\mu_{V_1} + \mu_{V_2}}\right)u \wedge \left(\overline{\mu_{V_1} + \mu_{V_2}}\right)v - \left(\mu_{E_1} + \mu_{E_2}\right)uv$$

$$= \left(\mu_{V_1}\right)u \wedge \left(\mu_{V_1}\right)v - \mu_{E_1}(uv = \mu_{V_1}(u) \wedge \mu_{V_1}(v) - \mu_{E_1}(uv) = \overline{\mu_{E_1}}(uv) = (\overline{\mu_1} \oplus \overline{\mu_1})uv$$

$$\overline{\left(\sigma_{E_1} + \sigma_{E_2}\right)uv} = \left(\overline{\sigma_{V_1} + \sigma_{V_2}}\right)u \wedge \left(\overline{\sigma_{V_1} + \sigma_{V_2}}\right)v - \left(\sigma_{E_1} + \sigma_{E_2}\right)uv$$

$$= \left(\sigma_{V_1}\right)u \wedge \left(\sigma_{V_1}\right)v - \sigma_{E_1}(uv)$$

$$= \sigma_{V_1}(u) \wedge \sigma_{V_1}(v) - \sigma_{E_1}(uv)$$

$$= \overline{\sigma_{E_1}}(uv)$$

$$= (\overline{\sigma_1} \oplus \overline{\sigma_1})uv$$

$$\overline{\left(\rho_{E_1} + \rho\right)uv} = \left(\overline{\rho_{V_1} + \rho_{V_2}}\right)u \wedge \left(\overline{\rho_{V_1} + \rho_{V_2}}\right)v - \left(\rho_{E_1} + \rho_{E_2}\right)uv$$

$$= \left(\rho_{V_1}\right)u \wedge \left(\rho_{V_1}\right)v - \rho_{E_1}(uv) = \rho_{V_1}(u) \wedge \rho_{V_1}(v) - \rho_{E_1}(uv) = \overline{\rho_{E_1}}(uv) = (\overline{\rho_1} \oplus \overline{\rho_1})uv$$

And similarly we can provethe theorem in following different cases

II when $uv \in E^*_1 \cap E^*_2$and $u, v \in V^*_1 \cap V^*_2$

IIIwhen $uv \in E'^*$i.e. $u \in V^*_1$and $v \in V^*_2$

IVwhen $uv \in E^*_1$and e. $u \in V^*_1$and $v \in V^*_1 \cap V^*_2$

4. Conclusion

Graph theory has many applications in solving various problems of several domains, including networking, communication, data mining, clustering, image capturing, image segmentation, planning, and scheduling. However, in some situations, certain aspects of a graph-theoretical system may be uncertain. Use of fuzzy-graphical methods in dealing with ambiguity and vague notions is very natural. Fuzzy graph play important role in representation of many uncertain decision making problem in daily life e.g. number theory, coding theory, cryptography, computer science, operation research etc. To deal with such problems, a number of generalizations of fuzzy graph have appeared in literature. This study described the idea of picture fuzzy graphs, complement of picture fuzzy graphs, Ring sum operation of picture fuzzy graphs, isomorphism of picture fuzzy graphs based on picture fuzzy relation have mentioned with common vertices and edges are taken as condition. Further we have described Ring sum of two picture fuzzy graph is also a picture fuzzy graph and some independent results on picture fuzzy graph in different cases which are fuzzy version of classical graph theory.

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